

# Fractional Knapsack Problem

- \* Given materials of different values per unit volume and maximum amounts, find the most valuable mix of materials which will fit in a knapsack of fixed volume.
- \* Two versions - 0/1 knapsack problem  $\Rightarrow$  Items cannot be divided  
fractional " "  $\Rightarrow$  Items can be divided
- \* Given a set of items, each with a weight & a value, determine a subset of items to include in a collection, so that the total weight is less than or equal to a given limit & the total value is as large as possible.

## \* Fractional knapsack

\* Items can be broken into smaller pieces. The problem statement is

$\rightarrow$  There are  $n$  items in the store.

$\rightarrow$  Weight of  $i^{\text{th}}$  item,  $w_i > 0$

$\rightarrow$  Profit of  $i^{\text{th}}$  item,  $P_i > 0$

$\rightarrow$  Capacity of the sack is  $W$

\* The items can be broken. So a fraction  $x_i$  of  $i^{\text{th}}$  item  $0 \leq x_i \leq 1$  can be taken.

\* Weight of  $i^{\text{th}}$  item is  $x_i \cdot w_i$  & profit of the item is  $x_i \cdot P_i$  that contributes to the sack.

\* Hence the objective of the algorithm is

$$\text{maximize } \sum_{i=1}^n x_i \cdot p_i \quad \text{subject to}$$

$$\sum_{i=1}^n (x_i \cdot w_i) \leq W$$

\* ALGORITHM

for  $i \leftarrow 1$  to  $n$  do

$$x[i] = 0$$

weight = 0

for  $i \leftarrow 1$  to  $n$  do

if weight +  $w[i] \leq W$  then

$$x[i] = 1$$

$$\text{weight} = \text{weight} + w[i]$$

else

$$x[i] = (W - \text{weight}) / w[i]$$

$$\text{weight} = W$$

break

return  $x$

\* Total time for this alg. is  $O(n \log n)$

Problem

Item	A	B	C	D	
Profit	280	160	120	120	give $W = 60$
weight	40	10	20	24	
$P_i/w_i$	7	10	6	5	

After sorting based on  $P_i/w_i$

Item	B	A	C	D
Profit	100	280	120	120
Weight	10	40	20	24
$P_i/w_i$	10	7	5	5

wt	profit
2	10
3	5
5	15
7	7
1	6
4	18
1	3

Find the optimal solution for the given knapsack problem.

Item	Profit	Weight	Remaining Weight
B	100	10	$60 - 10 = 50$
A	280	40	$50 - 40 = 10$
C	$\frac{10}{20} * 120$	10	$10 - 10 = 0$

$\therefore$  Solution set is  $\{1, 1, \frac{1}{2}, 0\}$

And the <sup>total</sup> profit of the sack =  $100 + 280 + \left[\frac{1}{2} * 120\right]$   
 $= 100 + 280 + 60$   
 $= 440$

# Job Sequencing with Deadlines

- \* The objective of the problem is to find ~~the~~ a sequence of jobs, which is to be completed within their deadline & gives maximum profit.
- \* In a set of 'n' given jobs which are associated with deadlines and profit is earned, if a job is completed by its deadline.
- \* These jobs need to be ordered in such a way, that there is maximum profit.

\* Eg:

Job	J <sub>1</sub>	J <sub>2</sub>	J <sub>3</sub>	J <sub>4</sub>	J <sub>5</sub>	J <sub>6</sub>
Deadline	2	1	3	2	1	
Profit	60	100	20	40	20	

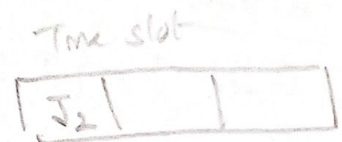
Sol<sup>n</sup> Sort the jobs in descending order of their profit

Job	J <sub>2</sub>	J <sub>1</sub>	J <sub>4</sub>	J <sub>3</sub>	J <sub>5</sub>
Deadline	1	2	2	3	1
Profit	100	60	40	20	20

∴ max d<sub>man</sub> = 3, n = 5

Select job J<sub>2</sub>

deadline = 1 ∴ timeslot (J<sub>2</sub>) = 1



Select job J<sub>1</sub>

deadline = 2 ∴ timeslot (J<sub>1</sub>) = 2



Select job J<sub>4</sub>

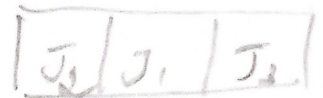
deadline = 2

timeslot 1 & 2 occupied so reject the job

Select job  $J_3$

deadline = 3

$\therefore$  timeslot ( $J_3$ ) = 3



All the slots filled.

$\therefore$  The required job sequence is  $J_2 \rightarrow J_1 \rightarrow J_3$

with maximum profit =  $100 + 60 + 20 = 180$