Fractional Knapsack Problem

* Given materials of different values per unit volume and maximum amounts, find the most valuable mix of materials which will fit in a knapsack of fixed volume.

* Two versions – 0/1 knapsack problem ⇒ Items cannot be divided
  fractional " " ⇒ Items can be divided

* Given a set of items, each with a weight & a value, determine a subset of items to include in a collection, so that the total weight is less than or equal to a given limit & the total value is as large as possible.

* Fractional knapsack
  * Items can be broken into smaller pieces. The problem statement is
    → There are n items in the store.
    → Weight of i^{th} item, $w_i > 0$
    → Profit of i^{th} item, $P_i > 0$
    → Capacity of the sack is W

* The items can be broken. So a fraction x_i of i^{th} item $0 \leq x_i \leq 1$ can be taken.

* Weight of i^{th} item is $x_i \cdot w_i$ & profit of the item is $x_i \cdot P_i$ that contributes to the sack.

* Hence the objective of the algorithm is

$$\text{maximize} \sum_{i=1}^{n} x_i \cdot P_i \quad \text{subject to}$$

$$\sum_{i=1}^{n} (x_i \cdot w_i) \leq W$$

* ALGORITHM

```
for i <- 1 to n do
        x[i] = 0

weight = 0

for i <- 1 to n do
   if weight + w[i] <= W then
            x[i] = 1
            weight = weight + w[i]

   else
         x[i] = (W - weight) / w[i]

         weight = W
         break

return x
```

* Total time for this alg. is $O(n \log n)$

Problem

Item	A	B	C	D
Profit	280	100	120	120
weight	40	10	20	24
P_i/w_i	7	10	6	5

give $W = 60$

$W = 15$

Find the optimal solution for the given knapsack problem.

wt	profit
2	10
3	5
5	15
7	7
1	6
4	18
1	3

After sorting based on P_i/w_i

Item	B	A	C	D
Profit	100	280	120	120
Weight	10	40	20	24
P_i/w_i	10	7	6	5

Item	Profit	Weight	Remaining Weight
B	100	10	$60 - 10 = 50$
A	280	40	$50 - 40 = 10$
C	$\frac{10}{20} * 120$	10	$10 - 10 = 0$

$\therefore$ Solution set is $\{1, 1, \frac{1}{2}, 0\}$

and the total profit of the sack $= 100 + 280 + [\frac{1}{2} * 120]$

$= 100 + 280 + 60$

$= 440$

Job Sequencing with Deadlines

* The objective of the problem is to find ~~those~~ a sequence of jobs, which is to be completed within their deadline & gives maximum profit.

* In a set of 'n' given jobs which are associated with deadlines and profit is earned, if a job is completed by its deadline.

* These jobs need to be ordered in such a way, that there is maximum profit.

* eg:

Job	J_1	J_2	J_3	J_4	J_5	~~J_6~~
Deadline	2	1	3	2	1	
Profit	60	100	20	40	20	

Soln: Sort the jobs in descending order of their profit

Job	J_2	J_1	J_4	J_3	J_5
Deadline	1	2	2	3	1
profit	100	60	40	20	20

Here $d_{max} = 3$, $n = 5$

Time slot

J_2		

Select job J_2

deadline $= 1$ $\therefore$ time slot $(J_2) = 1$

Select job J_1

J_2	J_1	

deadline $= 2$ $\therefore$ time slot $(J_1) = 2$

Select job J_4

deadline $= 2$ time slot 1 & 2 occupied so reject the job

Select job J_3

deadline = 3 $\therefore$ time slot (J_3) = 3

J_2	J_1	J_3

All the slots filled.

$\therefore$ The required job sequence is $J_2 \rightarrow J_1 \rightarrow J_3$

with maximum profit = 100 + 60 + 20 = 180